6.3 Exponential Functions

Ex 1: Use a Calculator to evaluate. Round to 3 places as needed.

A.) $64^{\frac{2}{3}}$ $= 64 \wedge (2/3) = 16$

B.) $3^{1.735}$ ≈ 6.727

C.) $158\left(\frac{5}{6}\right)^{3.275}$ ≈ 86.964

Definition of an Exponential Function

An **exponential function** is a function of the form

$$f(x) = Ca^x$$

where a is a positive real number ($a > 0$), $a \neq 1$, and $C \neq 0$ is a real number. The domain of f is the set of all real numbers. The base a is the **growth factor,** and because $f(0) = Ca^0 = C$, we call C the **initial value.**

EX: You have 6 rabbits and their population is tripling each year. How many will there be in 5 years?

$6(3)^5 = 1458$

How many in x years? $f(x) = 6(3)^x$

Example 2: Given a Description, Find the Exponential Function

Determine the exponential function whose graph is given.

$f(x) = C \cdot a^x$

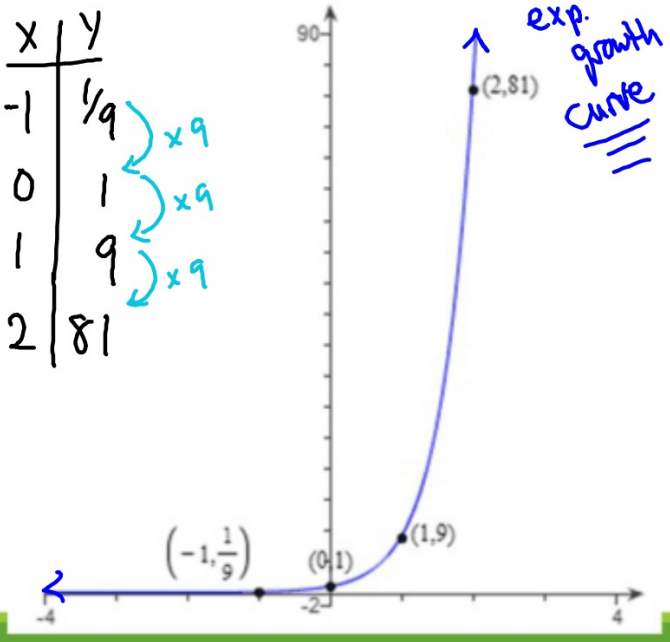
y-int

mult. pattern

$f(x) = 1(9)^x = 9^x$

x	y
-1	1/9
0	1
1	9
2	81

×9 ×9 ×9



Example 3: Identifying Linear or Exponential Functions

Determine whether the given function is linear, exponential, or neither. For those that are linear, find a linear function that models the data. For those that are exponential, find an exponential function that models the data.

(a) Linear

x	y
−1	5
0	2
1	−1
2	−4
3	−7

$m = -3$

$y = -3x + 2$

(b) Exponential

x	y
−1	32
0	16
1	8
2	4
3	2

$a = \frac{1}{2}$

$y = 16\left(\frac{1}{2}\right)^x$

(c) neither

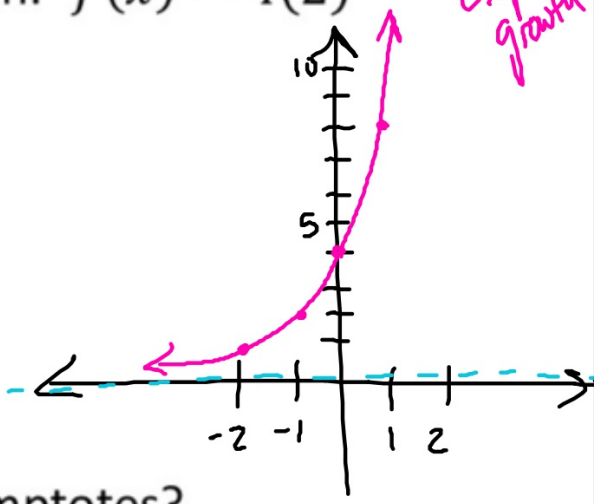
x	y
−1	2
0	4
1	7
2	11
3	16

Example 4: Graphing an Exponential Growth Function

Graph the exponential function: $f(x) = 4(2)^x$

exp. growth

X	Y
-2	1
-1	2
0	4
1	8
2	16



What is the Domain? Range?

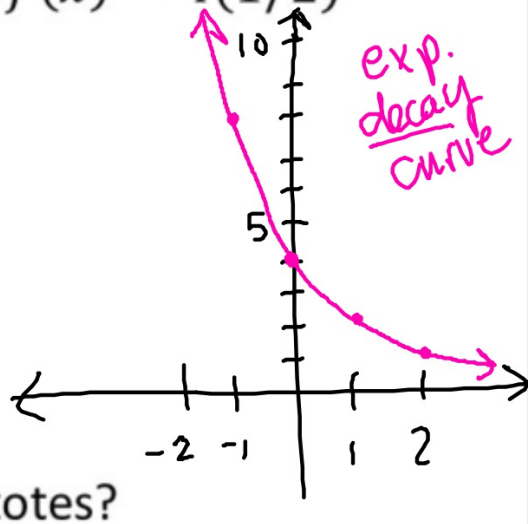
What are the Intercepts? Asymptotes?

D: all reals R: $(0,\infty)$ y-int: $(0,4)$ HA @ $y=0$

Example 5: Graphing an Exponential Decay Function

Graph the exponential function: $f(x) = 4(1/2)^x$

X	Y
-2	16
-1	8
0	4
1	2
2	1

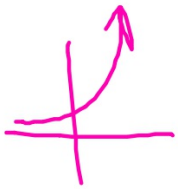


What is the Domain? Range?

What are the Intercepts? Asymptotes?

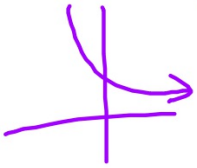
D: all reals R: $(0, \infty)$ y-int: $(0, 4)$ HA @ $y = 0$

$f(x) = C a^x \quad \begin{cases} a \neq 1 \\ a > 0 \end{cases}$



If $a > 1$, then it is exp. growth

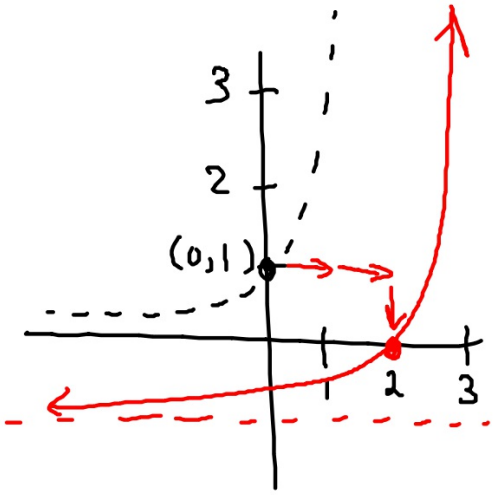
If $0 < a < 1$, then it is exp. decay



Example 6: Graphing an Exponential Function

Graph $f(x) = 4^{x-2} - 1$ and determine the domain, range, and horizontal asymptote.

shift right 2 and down 1



D: all reals

R: $(-1, \infty)$

HA: $y = -1$

Euler's Number

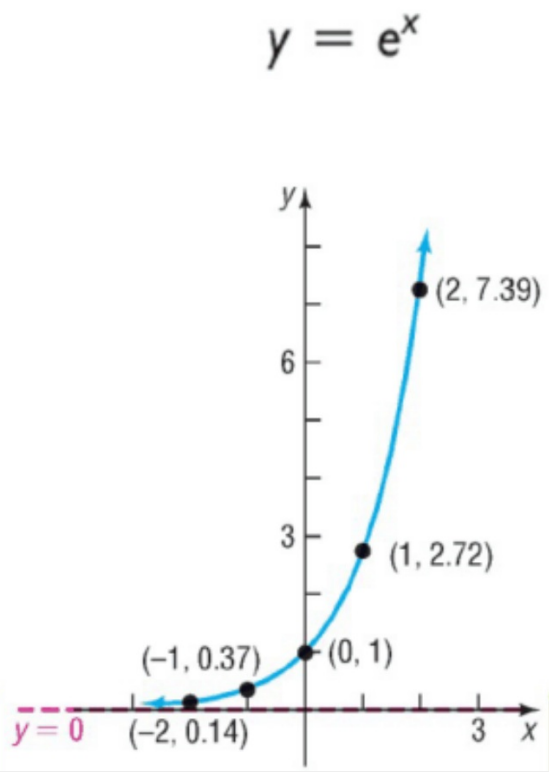
The number *e* is known as the natural base. It is named after Swiss mathematician Leonhard Euler, who discovered it. The number *e* is used in population models as well as in financial models.

$$e \approx 2.71828183\ldots$$

Note: *e* is an irrational number (like π)

Graph of the Exponential Function Base *e*

x	e^x
-2	$e^{-2} \approx 0.14$
-1	$e^{-1} \approx 0.37$
0	$e^{0} \approx 1$
1	$e^{1} \approx 2.72$
2	$e^{2} \approx 7.39$



Example 8: Solve exponential functions

* make each side have the same base

A. $8^{-x+14} = 16^{x}$

$(2^3)^{-x+14} = (2^4)^{x}$

$2^{-3x+42} = 2^{4x}$

$-3x+42 = 4x$

$42 = 7x$

$6 = x$

B. $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

$e^{-x^2} = e^{2x} \cdot e^{-3}$

$e^{-x^2} = e^{2x-3}$

$-x^2 = 2x-3$

$0 = x^2+2x-3$

$0 = (x-1)(x+3)$

$x = 1, -3$

Your Turn

$5^{x^2+8} = 125^{2x}$

$5^{x^2+8} = (5^3)^{2x}$

$5^{x^2+8} = 5^{6x}$

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2, 4$

Example 9: $D(h) = 8e^{-0.55h}$ represents the number of milligrams, *D*, of a drug in a patient's bloodstream after *h* hours.

A) How many mg are present after 1 hour?

$$D(1) = 8e^{-.55} \approx 4.616 \text{ mg}$$

B) After 12 hours?

$$D(12) = 8e^{-.55(12)} \approx 0.011 \text{ mg}$$